ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEM INCLUDING TRANSMISSION LOSSES

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DEDICATED

TO

MY LATE FATHER

CERTIFICATE

Certified that this work "Economic Load Scheduling of Hydro-Thermal System Including Transmission Losses" by K.N. Srivastava has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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K. N. SRIVASTAVA

ABSTRACT

In an interconnected power system, if the installed capacity exceeds total demand plus losses, more than one operating strategy exists to meet the load demand in the system and hence it is important to seek the best one that results in the minimum operating cost. Economic load scheduling of hydro-thermal system is much more complex than that of a purely thermal system, for two reasons. Firstly, there is negligible operating cost associated with hydro stations. Secondly, the available water resources at hydro stations in a given period are limited. Several nonlinear programming techniques have been used in the past to solve this problem. But, the basic requirements of any algorithm for implementing optimum operation strategies in integrated power systems are speed, efficiency and reliable convergence. In view of these factors, successive linear programming technique is a powerful candidate. In the present work, the application of successive linear programming technique is made to solve short range scheduling problem of a hydro-thermal system. Considerable reduction in storage and computational efforts, is observed.

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CHAPTER 1

INTRODUCTION

With the advent of the high speed digital computers, the various problems that arise in the power system, have been solved to a very great extent. The early application of the digital computers, however, was restricted to the problems of analysis such as load-flow, short circuit and stability studies with greater emphasis on realistic system modelling. The economic load scheduling of a large scale interconnected power system, is also one of the power system problems that a computer handles in a rigrous manner using sophisticated mathematical model for its various components. In an interconnected power system with several sources,more than one operating strategy is possible to meet the load demand in the system and hence it is natural for the operator to seek the best operating strategy that results in the minimum operating costs. This is achieved by optimising a pre-specified operating criterion which is the cost of fuel at generating stations, with reference to some control variables (plant generations, transformertaps etc.) in the system. This is usually referred to as the economic load scheduling, and has been the concern of the power engineers for a long time.

A typical power system network consists of a large number of thermal and hydro plants connected to the different

load centres through lossy transmission networks. An important objective in the operation of such a power system is the generation and transmission of power to meet the system load demand at each instant of time with the minimum fuel cost. The study of the optimisation problem is of paramount importance in a developing country like India with interconnected grid systems having both the hydel as well as the thermal plants to meet load demands.

Hydro-thermal optimal load scheduling is certainly more exciting and complex problem than that of purely thermal system optimisation problem, for two reasons. Firstly, there is negligible operating cost associated with hydro stations and hence the hydro generations are not decided by cost considerations alone. Secondly, the available water resources at hydro stations, in a given period of time, are limited and this poses an extra constraint on hydro generations. In view of these factors, the load scheduling of hydro-thermal systems becomes an interval problem, the attempt being to determine generations over an interval of time so as to minimise the cost of generation in that interval, meeting at the same time, the load demand and other constraints. Depending upon the duration of the period of optimisation, the problem is referred to as short range or long range problem. Such a distinction becomes essential mainly because of the available information about the system.

The earliest formulation and statement of optimality conditions for the economic scheduling of hydro-thermal system date back to the pioneering work of Chandler, Dandeno, Glimn and Kirchmayer [3] . This uses Lagrangian multipliers to append the equality constraints to the objective function. The unconstrained optimisation of the augmented objective function results in solving a set of equations together with the equality constraints. Dandeno [4] observed, while working with these coordination equations on the digital computer, that solutions were sometimes obtained that dictate generations beyond the plant capacity and also negative generation of certain plants since the constraints regarding the plant capacities, were not incorporated in the initial formulation.

Wije Perera et. al. [5] imposes the upper and lower bounds on the operating range of each plant in the form of inequality constraints and solved the problem, by using Pontryagin's Maximum Principle followed by Kuhn-Tucker conditions.

Sokkappa [27] formulated the hydro-thermal scheduling problem as a nonlinear programming problem. Transmission losses were considered through the use of loss coefficients. For each subinterval of optimisation, the constraint that is most likely to be violated, was picked up and a slack variable

was associated with this constraint. The gradient of the objective function i.e. cost of the generation, was evaluated and the steepest descent method was used to obtain the solution of the problem, starting from a known initial schedule. Such a procedure becomes inefficient when the dimension of the problem is large.

Gopala Rao [11] has formulated the short range problem as an additively separable nonlinear programming problem and used Lasdon's decomposition technique [28], to split the problem of higher dimension into sub-problems of smaller dimensions to be solved iteratively. The main difficulty in this method is the initial choice of the dual variable vector which plays a dominant role in the computational time required.

Dynamic programming technique was used by Singh and Aggarwal [26] to solve a short range problem. Hano et.al.[29] have used continuous maximum principle of Pontryagin to the economic operation for hydro-thermal systems. Oh [30] has employed the discrete maximum principle for the solution of the same problem. In all these methods, the computational requirements for a realistic size problem become enormously high.

OUTLINE OF THE THESIS

The central theme of the present work is the application of the successive dual linear programming technique to obtain

solutions to the complex problem of optimal scheduling in hydro-thermal power systems including the transmission losses. A brief outline of the work reported in the different chapters is given below.

The second chapter introduces the linear programming problem and the simplex method due to George B. Dantzig for its solution. The relevance of the dual problem has been discussed along with the primal-dual relationship. A brief account of the various nonlinear programming techniques has also been given. Nonlinear minimisation via linear approach has been discussed along with the algorithm.

The chapter 3 briefly describes the overview of the earlier work done in the field of economic load scheduling of the hydro-thermal system. A few of the existing formulations have been discussed briefly.

In chapter 4, the formulation and solution procedure for the economic load scheduling problem have been presented. The nonlinear equations involved in the formulation have been suitably linearised around a nominal schedule. Successive dual linear programming technique is used to solve the resulting linear programming problem. A numerical example

is solved to illustrate the solution procedure for a three-thermal-one-hydro system. Transmission losses have been considered through the use of loss coefficients. Considerable reduction in storage and computation effort is observed.

Finally chapter 5 concludes with the overall summary of the work reported in this thesis. A few suggestions or possibilities for further work in this area are also given.

CHAPTER 2

OPTIMIZATION TECHNIQUES

2.1 INTRODUCTION

The optimal load scheduling of a hydro-thermal system is essentially a mathematical programming problem. It is for this reason it becomes imperative to study the optimization techniques for solving the mathematical programming problems.

The mathematical programming techniques referred to as optimization techniques are useful in finding the optimal of a function known as objective (or goal) function of several variables under a prescribed set of constraints. Based on the nature of the equations involved, the problem can be classified as linear or non-linear programming problem. A linear programming problem (LP or LPP) is one in that the objective function and the constraints, both are linear function of the decision variables. If any one of the functions among the objective and constraint equationshappen to be non-linear, the problem is referred to as a non-linear programming (NLP) problem.

2.2 LINEAR PROGRAMMING (LP)

The linear programming type problem was first recognised in the 1930s by economists while developing methods for the optimal allocation of resources. In 1947, George B. Dantzig [6] gave the simplex method for the solution of LP.

Since then much progress has been made in the theoretical development and also in the practical application of LP. Amongst these, the theoretical contribution made by Kuhn and Tucker had led to the development of the duality theory in LP [9,10].

A standard form of LP enables one to use a standard algorithm. All LPs can be cast into the following form.

Find the $X \equiv (x_1, x_2, \ldots, x_n)$, that minimizes*

The function, $F(X) = \sum_{j=1}^{n} c_j x_j$ .. (2.1a)

subject to the constraints,

$$\sum_{j=1}^{n} a_{ij} x_j = b_i , \quad i = 1, \ldots, m. \qquad \text{.. (2.2a)}$$

$$x_j \geq 0, \quad j=1, \ldots n \qquad \text{.. (2.3a)}$$

In the matrix form, LP can be written as,

Find X, that minimizes.

$F(X) = C^T X,$ .. (2.1b)

Subject to

$$AX = B \qquad \text{.. (2.2b)}$$

and, where a matrix or vector inequality is taken to apply to each element of the matrix or vector separately,

$$X \geq 0 \qquad \text{.. (2.3b)}$$

where A is the m x n matrix.

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*Maximisation of a function is the minimisation of negative of the function.

In all cases of interest, $m < n$ because of $m=n$ and none of the equations are redundant, there is only solution to $AX=B$. If $m > n$, then there would be $m-n$ redundant equations which could be eliminated. With $m < n$, the system is undete.. mined and, in general possessesan infinite number of solutions of which the optimal one is sought. The non-negativity constraints $X \geq 0$ are the only inequalities present in the standard form.

In order to put an arbitrary problem into the standard form, two things are done:

(a) Inequalities are converted into equalities. If a constraint appears in the form of a "less than" type of inequality as

$$a_{k1} x_1 + a_{k2} x_2 + ---- + a_{kn} x_n \leq b_k$$

it can be converted into the equality form by adding a non-negative slack variable x_{n+1} as follows,

$$a_{k1} x_1 + a_{k2} x_2 + ---- + a_{kn} x_n + x_{n+1} = b_k$$

Similarly, the nonnegative surplus variable is subtracted to convert a "greater than type" of inequality into the equality form.

(b) Variables are transformed, so that, they are all nonnegativ If the variables in the original problem are not required to be non-negative, they can be written as the difference between

two non-negative variables :

$$x_j = x_j' - x_j'' , \qquad .. (2.4)$$

where $x_j' \geq 0$ and $x_j'' \geq 0$. The new nonnegative variables would appear in objective function as well as in constraints. But the slack or/and surplus variables would appear only in the constraints.

In connection with LP a number of standard definitions are as under.

(a) A feasible solution is one that satisfies Eq. (2.2) and inequalities Eq. (2.3)

(b) A basic feasible solution is a feasible solution with no more than m non-zero x_j. In other words, it has at least (n-m) x_j that are zero. The feasible solution x_j associated with the basis vector A_j is known as basic feasible solution.

(c) A nondegenerate basic feasible solution has exactly m positive x_j.

(d) A optimal basic solution is a basic feasible solution for which objective function is optimal.

The simplex method of Dantzig [6] is a powerful scheme to get a basic feasible solution. It makes use of simplex algorithm[6,7,8]. Assuming that the problem is feasible

and has a basic feasible solution readily available, we note that if any of cost coefficients c_t corresponding to a non-basic variable is negative, the objective function will be reduced if we increase the associated variable from zero. Therefore, to improve the basis, we can bring any such x_t into the basis. If more than one c_t is negative, we can choose among them, and the criterion most used today is to choose the 't' for which

$$c_t = \min_j [c_j] \qquad .. (2.5)$$

This choice selects the variable for which the objective function reduces at the greatest rate. It should be noted that some other choice may occasionally produce a greater reduction in the objective function for this step since, because of the constraints, the alternative variable may be capable of more change. However, the above criterion is usually the best, because it is simplest and also because experience has shown that it is overall efficient.

If $c_t = \min [c_j]$ is non-negative, there is no variable that can be increased to reduce the objective function; this situation is called the optimality condition.

We now know which variable to bring into the basis (assuming that some $c_t < 0$) and we find, that, we must eliminate

the variable x_s for which,

$$\frac{b_s}{a_{st}} = \min_{a_{it} > 0} [b_i/a_{it}] \qquad .. (2.6)$$

and so, we have the simplex algorithm for proceeding from any basic feasible solution to an optimal basic solution if one exists. When all the a_{st} in the column corresponding to a negative c_t are also negative we can increase x_t indefinitely without causing any variable to become negative. This is the case when the problem has an unbounded solution.

The simplex method is a two phase algorithm : phase I is to find a basic feasible solution if it is not readily available and phase II is to find the optimal solution, out of basic feasible solutions which are finite. The simplex method is described in the following steps.

(i) Phase I of the method : Arrange the system of Eqs. (2.2), so that, all b_i are positive or zero by changing, where necessary, the signs on both sides of any of the equations.

(ii) Introduce to this system a basic set of artificial variables, $y_1, y_2, \ldots, y_m$ where each $y_i \geq 0$, so that, it becomes

$$\begin{aligned} a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n + y_1 &= b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n + y_2 &= b_2 \\ &\vdots \\ a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n + y_m &= b_m \\ b_i \geq 0 & \end{aligned} \qquad .. (2.7)$$

The objective function of Eq. (2.1) can be written as

$$c_1 x_1 + c_2 x_2 + \dots + c_n x_n + (-f) = 0 \quad .. (2.8)$$

(iii) Define a quantity w as the sum of the artificial variables

$$w = y_1 + y_2 + \dots + y_m \quad .. (2.9)$$

and use the simplex algorithm to find $x_i \geq 0$ and $y_i \geq 0$ which minimises the 'w' and satisfy the Eqs. (2.7) and (2.8). From Eqs. (2.7) and (2.9) we get,

$$w = d_1 x_1 + d_2 x_2 + \dots + d_n x_n + w_o \quad .. (2.10)$$

where

$$d_i = -(a_{1i} + a_{2i} + \dots + a_{mi}) \quad .. (2.11)$$

$$\text{for } i = 1, 2, \dots, n$$

and

$$w_o = (b_1 + b_2 + \dots + b_m) \quad .. (2.12)$$

In the algorithm, Eq. (2.10), is used together with the Eqs. (2.7) and (2.8) for the optimisation.

(iv) 'w' is called the infeasibility form and has the property that if, as a result of phase I, minimum of $w > 0$, then no feasible solution exists for the original LP problem. On the other hand, if minimum of $w=0$, then the resulting solution will act as the starting point for the phase II.

(V) Phase II of the method : Apply the simplex algorithm to the adjusted system of equations at the end of phase I to obtain a solution, if a finite one exists which optimises the value of objective function. The flow chart is given in the appendix.

Associated with every LP problem, called the primal, there exists another LP problem called its dual. The general primal-dual relations and the correspondence rules to be applied in deriving the general primal-dual relations are given table (2.1) and table (2.2) respectively [8].

Table 2.1

Primal problem	Corresponding Dual Problem
Minimise $f = \sum_{j=1}^{n} c_j x_j$ subject to	Maximise $v = \sum_{i=1}^{m} y_i b_i$ subject to
$\sum_{j=1}^{n} a_{ij} x_j = b_i$, $i=1,2,\ldots m_1$	$\sum_{i=1}^{m} y_i a_{ij} = c_j$, $j=n_1+1, n_1+2, \ldots m$
$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$, $i=m_1+1, m_1+2, \ldots m$	$\sum_{i=1}^{m} y_i a_{ij} \leq c_j$, $j=1,2, \ldots n_1$
where	where
$x_j \geq 0$, $j=1,2, \ldots n_1$	$y_i \geq 0$, $i = m_1+1, m_1+2, \ldots m$
and	and
x_j unrestricted in sign, $j = n_1+1, n_1+2, \ldots, n$	y_i unrestricted in sign, $i=1,2, \ldots m_1$

Table 2.2

Primal quantity	Corresponding dual quantity
Objective Function : Minimise C^TX	Objective Function : Maximise Y^TB
Variable $x_i \geq 0$	i^{th} constraint $Y^TA_i \leq c_i$ (inequality)
Variable x_i unrestricted in sign	i^{th} constraint $Y^TA_i = c_i$ (equality)
j^{th} constraint, $A_jX = b_j$ (equality)	j^{th} variable y_j unrestricted in sign
j^{th} constraint, $A_jX \geq b_j$ (inequality)	j^{th} variable $y_j \geq 0$
Coefficient matrix $A = \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_m \end{bmatrix}$	coefficient matrix $A^T = [A_1, A_2, \ldots A_m]$
Right hand side vector B	right hand side vector C
Cost coefficients C	cost coefficients B

Since, in general, an additional constraint requires more computational efforts than an additional variable in LP, it is evident, that, it is computationally more efficient to solve the dual problem, in case the primal problem has more number of constraints that its dual. This is one of the advantages of the dual problem.

2.3 NONLINEAR PROGRAMMING PROBLEM

A non-linear programming problem may be constrained or unconstrained. In most of the cases, the problems are constrained one. Some of the most powerful and convenient methods of solving constrained optimisation problems involve the transformation of the problem into one of unconstrained type.

The constraints involved in NLP may be linear or non-linear. For problems having linear constraints, large step gradient procedures are available like gradient projection method of Rosen [19]. For problems with non-linear constraints, a set of necessary conditions to be satisfied by the solution is obtained by Kuhn and Tucker [20]. Making use of these conditions, attempts are made to obtain the solution. In order to point out some of the difficulties in such procedures, a little more discussion of these conditions is necessary. Peschon [21] has applied a gradient technique to satisfy Kuhn and Tucker conditions arising in the reactive power optimisation problem.

A computationally better procedure what is known as the sequential unconstrained minimisation techniques (SUMT), has been used by Gopala Rao[11] and Sasson [22]. Briefly their approaches are discussed below.

The method of Fiacco and McCormick [24] is based on the transformation,

$$F(X, r_k)=f(X)-r_k \sum_{i=1}^{m} \frac{1}{g_i(X)} + \frac{1}{\sqrt{r_k}} \sum_{j=1}^{p} l_j^2 (X) \qquad .. (2.13)$$

where $f(X)$ is the objective function, $g_i(X) \leq 0$, $i=1, .. m$ are inequality constraints, $l_j(X)=0$, $j=1, ... p$ are equality constraints and r_k is the penality parameter.

A set of decreasing r_k values are selected and $F(X,r_k)$ is minimised for each one of them. When r_k is zero (i.e. sufficiently small), minimum of 'F' approaches the minimum of 'f'. Details can be had from Ref. [23, 24] .

Gradient methods are widely used for NLP problems. Many of the commonly used gradient methods start with an initial guess, search for a stationary point by making changes in the decision variables in proportional to measured partial derivatives to obtain the next guess and so on. The choice of step size is critical and the so-called 'optimum gradient' [14] is the best choice. A few recent techniques of static optimisation are also compared in Ref. [16] . Other gradient techniques like parallel tangents (PARTAN), conjugate gradient [18], Fletcher-Powell technique [17], etc. use the gradient and some other information to find a new direction.

2.4 LINEARISATION APPROACHES TO NONLINEAR PROBLEMS

An attractive approach to some non-linear programming problems is to linearise them in a way that permits the use of LP methods. The convergence problem and high solution time make NLP techniques not very useful. A successive LP formulation is proposed to correct a potentially vulnerable system so as to bring it to an optimal normal state. This approach has been used by Paranjothi [25] for optimal power flow problem, and is also used in the present work for the economic load scheduling of hydro-thermal system. Handling equality constraints in LP is not as difficult as in NLP.

Consider the original NLP problem;

Minimise

$$f(X), \qquad ..(2.14)$$

Subject to

$$g_i(X) \leq 0 , \quad i=1, \ldots m \qquad ..(2.15)$$

and

$$l_j(X) = 0 , \quad j=1, \ldots p \qquad ..(2.16)$$

Let X_o be a solution point which may be feasible or infeasible. Expand equations (2.14), (2.15) and (2.16) by Taylor series expansion.

$$f(X) \doteq f(X_o) + \nabla f(X_o)^T(X-X_o) \equiv F(X) \qquad .. (2.17)$$

$$g_i(X) \doteq g_i(X_o) + \nabla g_i(X_o)^T(X-X_o) \equiv G_i(X) \qquad .. (2.18)$$

$$l_j(X) \doteq l_j(X_o) + \nabla l_j(X_o)^T(X-X_o) \equiv L_j(X) \qquad .. (2.19)$$

The corresponding LP problem is as follows :

Find X that minimises F(X),

subject to

$$G_i(X) \leq 0 \; ; \qquad i=1,2, \ldots m$$

and

$$L_j(X) = 0 \; ; \qquad j=1,2, \ldots p$$

The functions are again linearised about the new point, thus obtained, and the process is continued until the desired accuracy in the objective function value is observed. The algorithm can be as under :

(i) Select a starting point.

(ii) Linearise the functions about this point by Taylor series expansion.

(iii) Formulate corresponding LP problem

(iv) Solve LP problem so as to obtain a new better point

(v) Check whether the new point is optimal. If so, stop the process. Otherwise, go to step (ii) and repeat onwards.

CHAPTER 3

ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEMS

3.1 INTRODUCTION

The problem of economic load scheduling of hydro-thermal systems can be broadly classified into long and short range problems. The long range problem refers normally to an annual problem. The inflows into the reservoirs show an annual cyclicity. Furthermore, there may be a seasonal variation in power demand on the system. This also shows an annual cyclicity. The optimisation interval of one year duration is thus a good choice for long range optimal generation scheduling studies. In this case, the solution to the scheduling problem consists of determination of the ammounts of water to be drawn from the reservoirs for hydro-generation in each scheduling subinterval and corresponding thermal generations to meet the load demand over each subinterval utilising the entire amount of water available for power generation during the total interval.

The short range problem which is the objective of the present work, has an optimisation interval of a day or a week. This period is normally subdivided into hourly or half hourly subintervals for scheduling purposes. The solution to the problem will stipulate the amounts of water to be utilised over each day or week (the scheduling subinterval of the long

range problem). Then, the solution to the short range problem consists of an optimal plan for utilisation of this water for power generation and the corresponding optimal thermal generations are determined considering the load demand on the system and the constraints imposed on its operation.

Several attempts have been made in the past to solve the economic load scheduling problem of hydro-thermal systems using techniques such as Dynamic programming, Pontryagin's maximum principle, NLP techniques etc. In the following section, a brief overview of some of them is presented.

3.2 OVERVIEW OF THE ECONOMIC LOAD SCHEDULING PROBLEM

The earliest formulation is based upon the short range hydro-thermal economic optimisation problem developed by Chandler, et.al. [3]. The main cost of operation of the hydro-thermal system is the cost of fuel used in the thermal plants as the cost of water in hydro plant is negligible compared to the cost of the fuel of the thermal plants. Hence, the objective is to determine the generation of individual plants, both thermal as well as hydel such that, the cost of total generation is the minimum and at the same time, total demand and losses are continuously met. As it is a short range problem, there

will not be any appreciable change in the level of water in the reservoirs during the optimisation interval (i.e. effects of evaporation and rain fall are ignored) and hence the variation in head of the reservoir of the hydel plants will not be crucial during the interval. However, specified quantity of water Y_i must be utilised over the interval at i^{th} hydel plant. The problem is formulated as follows :

Find the thermal generation S_j and the hydel generation h_i which are the function of time, such that the cost function as defined by,

$$C_T = \int_0^T \left\{ \sum_{j=1}^{N} C_j(S_j(t)) \right\} dt \qquad .. (3.1)$$

is minimum, subject to the equality constraint,

$$\sum_{j=1}^{N} S_j(t) + \sum_{i=1}^{M} h_i(t) = P_D(t) + P_L(S,h) \qquad (3.2)$$

i.e. at each instant of time, the total demand $P_D(t)$ and the total losses $P_L(S,h)$ which is the function of plant generations, is equal to the total generation of the plants, and

$$\int_0^T \left\{ y_i \left(h_i(t)\right) \right\} dt = Y_i \qquad \text{.. (3.3)}$$

i.e. in a given interval T, specified amount of water Y_i is utilised at i^{th} hydel plant. The different symbols are as,

$S_j(t)$ = Power generation of the j^{th} thermal plant

$h_i(t)$ = Power generation of the i^{th} hydel plant

T = Optimisation interval defining the short range problem

$C_j(S_j(t))$ = Fuel cost in Rs/hour of the j^{th} thermal plant which is a quadratic function of plant generation $S_j(t)$

$y_i(h_i(t))$ = Discharge of water in cubic ft./hour at the i^{th} hydel plant which is a function of hydel generation $h_i(t)$.

N = Number of thermal plants

M = Number of hydel plants

$P_D(t)$ = Load demand at any instant 't'.

$P_L(S,h)$ = Transmission losses which are function of plant generations.

The nonlinear objective function given by equation (3.1) along with the equality constraints of equations (3.2) and (3.3) are transformed into unconstrained optimisation with the

proper choice of the langrange multipliers λ and Υ_i. The following coordination equations are obtained for this optimisation problem.

$$\frac{dC_j}{dS_j} + \lambda\frac{\partial P_L}{\partial S_j} = \lambda \qquad ..(3.4)$$

$$\Upsilon_i\frac{dy_i}{dh_i} + \lambda\frac{\partial P_L}{\partial h_i} = \lambda \qquad ..(3.5)$$

for $j = 1, \ldots N$ and $i = 1, \ldots M$

The coordination equations (3.4) and (3.5), when solved together with equations (3.2) and (3.3), give the solution, for $S_j(t)$ and $h_i(t)$ over the entire interval. The values Υ_i (known as water value) which effectively convert incremental water discharge (i.e. $\frac{dy_i}{dh_i}$) as a function of plant generation into incremental plant costs, determine the amount of water to be utilised at each hydel plant. Therefore, Υ_i must be so chosen that the desired amounts of water are utilised, i.e. equation (3.3) is satisfied. It is observed that Υ_i is constant while λ is a function of time. Dandeno [4] while working with these coordination equation on the digital computer observed that the direct application of these equations results in solu-

tions that sometimes, dictate generations beyond plant capacities and also negative generations because the constraints regarding plant capacities are not included in the problem formulation.

Therefore, the problem is extended by imposing upper and lower bounds on the operating range of each plant as indicated by the inequalities (3.6) and (3.7)

$$S_j^{min} \leq S_j \leq S_j^{max} \quad , \quad j=1, \ldots N \qquad .. (3.6)$$

and

$$h_i^{min} \leq h_i \leq h_i^{max} \quad , \quad i=1, \ldots M \qquad .. (3.7)$$

where S_j^{min} and h_i^{min} are the minimum and S_j^{max} and h_i^{max} are the maximum limits of operation of the corresponding plant. By including the inequalities, Dandeno [4] doubted about the constancy of γ_i. The preliminary step in the extension is the application of Pontryagin's maximum principle by Wije Perera [5] . By applying this maximum principle followed by Kuhn-Tucker conditions, the problem reduces to maximising the Hamiltonian H at each instant of time i.e.

$$\text{Maximise} \quad H = -\sum_{j=1}^{N} C_j(S_j(t)) - \sum_{i=1}^{M} \gamma_i \, y_i(h_i(t)) \qquad .. (3.8)$$

The maximisation of H at each instant of time is an auxiliary problem of static type. The value of γ_i should be so chosen that the equation (3.3) is satisfied. γ_i actually corresponds to the price of water. An increase in γ_i would result in the lesser water usage at the i^{th} hydel plant and vice-versa. Maximisation of 'H' is equivalent to the minimisation of the function C_T of equation (3.8) as shown below :

$$\text{Minimise } C_T = \text{Maximise } (-(H))$$

$$= \sum_{j=1}^{N} C_j(S_j(t)) + \sum_{i=1}^{M} \bar{\gamma}_i \, y_i(h_i(t)) \quad .. \ (3.9)$$

The minimisation of the function C_T as defined by equation (3.9) corresponds to the minimisation of the total operating cost of the thermal as well as hydel plants. The first term in equation (3.9) represents operating cost of the thermal plant and the second term, that of the hydel plant which is almost negligible in practice.

Gopala Rao [11] formulated the economic load scheduling problem of hydro-thermal system as an additively separable nonlinear programming problem and used Lasdon's decomposition technique [28] for its solution. The optimisation interval is subdivided into 'K' equal intervals such that the load demand

at all the stations remains constant. To account for losses, A.C. power flow equations are used. Bus voltage magnitudes and angles constitute the problem variables. Maximum and minimum limits on active and reactive power generations and maximum limits on bus voltage magnitudes are imposed.

Let there be 'M' hydro and 'N' thermal stations and 'm' be the total buses. Then, in any k^{th} interval, the injection equations can be written as

$$I_i(\delta^k, E^k) = \sum_{j=1}^{m} E_i^k \; E_j^k \; Y_{ij} \; \mathrm{Cos}(\theta_{ij} - \delta_i^k + \delta_j^k) \qquad .. (3.10)$$

$$J_i(\delta^k, E^k) = -\sum_{j=1}^{m} E_i^k \; E_j^k \; Y_{ij} \; \mathrm{Sin}(\theta_{ij} - \delta_i^k + \delta_j^k) \qquad .. (3.11)$$

where E^k and δ^k are bus voltage magnitude and angle in k^{th} interval

$Y_{ij} \underline{/\theta_{ij}}$ is the ij^{th} element of Y bus, and I_i and J_i are injections into the system at i^{th} bus.

Then power generation equations will be

$$P_i^k(\delta^k, E^k) = I_i(\delta^k, E^k) + P_{Di}^k \qquad .. (3.12)$$

$$Q_i^k(\delta^k, E^k) = J_i(\delta^k, E^k) + Q_{Di}^k \qquad .. (3.13)$$

for $i = 1, \ldots, (M+N)$.

and $k = 1, \ldots, K$

where P_i^k and Q_i^k are active and reactive power generations at i^{th} bus, and P_{Di}^k and Q_{Di}^k are active and reactive loads at the same bus in the k^{th} interval.

Similarly, the load demand equations will be,

$$I_i(\delta^k, E^k) + P_{Di}^k = 0 \qquad .. (3.14)$$

$$J_i(\delta^k, E^k) + Q_{Di}^k = 0 \qquad .. (3.15)$$

for $i = M+N+1, \ldots m$

and $k = 1, \ldots, K$

The objective function, i.e. the cost of generation is given as ,

$$f(\delta^k, E^k) = \sum_{k=1}^{K} \sum_{i=M+1}^{M+N} C_i(P_i^k(\delta^k, E^k)) \qquad .. (3.16)$$

Now, the optimisation problem can be mathematically stated as,

Minimise $f(\delta^k, E^k)$

Subject to (i) load demand equations,

$$I_i(\delta^k, E^k) + P_{Di}^k = 0 \qquad .. (3.17)$$

$$J_i(\delta^k, E^k) + Q_{Di}^k = 0 \qquad .. (3.18)$$

for $i = M+N+1, \ldots, m$

and $k = 1, \ldots K$

(ii) engineering limits on the system

$$P_i^{min} \leq P_i^k(\delta^k, E^k) \leq P_i^{max} \qquad \text{.. (3.19)}$$

$$Q_i^{min} \leq Q_i^k(\delta^k, E^k) \leq Q_i^{max} \qquad \text{.. (3.20)}$$

$$E_i^k \leq E_i^{max} \qquad \text{.. (3.21)}$$

for $i = 1, \ldots M+N$

and $k = 1, \ldots K$

and (iii) coupling equations at hydro plants

$$\sum_{k=1}^{K} y_i(P_i^k(\delta^k, E^k)) = Y_i \qquad \text{.. (3.22)}$$

for $i = 1, \ldots M$

where Y_i is the volume of water available for power generation

The objective function can be rewritten as,

$$f(\delta^k, E^k) = \sum_{k=1}^{K} f_k'(\delta^k, E^k) \qquad \text{.. (3.23)}$$

where

$$f_k'(\delta^k, E^k) = \sum_{i=M+1}^{M+N} C_i(P_i^k(\delta^k, E^k)) \qquad \text{.. (3.24)}$$

Associating dual variables $u_1, \ldots u_M$ with coupling equations, the Lagrangian function can be written as,

$$L(\delta^k, E^k, u) = \sum_{k=1}^{K} f_k'(\delta^k, E^k) + \sum_{i=1}^{M} u_i \left(\sum_{k=1}^{K} \left\{ y_i(P_i^k(\delta^k, E^k)) - Y_i \right\} \right) \quad .. (3.25)$$

Now the dual problem will be,

Maximise $h(u)$ over all u

$$\text{where} \quad h(u) = \min_{\delta^k,\, E^k \in S^k} L(\delta^k, E^k, u) \quad .. (3.26)$$

$$\text{for} \quad k = 1, \ldots K$$

The set S^k (known as constraining set) to which the k^{th} subproblem variables ^{k} and E^k are to be constrained, is defined by the equations (3.17) to (3.21).

Using decomposition technique, the Lagrangian function can be minimised by solving the following 'K' subproblems

$$\text{for} \quad k = 1, \ldots K$$

$$\min f_k'(\delta^k, E^k) + \sum_{i=1}^{M} u_i\, y_i(P_i^k(\delta^k, E^k))$$

subject to (i) the load demand equations of (3.17) and (3.18)

and (ii) the engineering limits of equation (3.19) to (3.21), on the system.

To sum up, an additively separable nonlinear programming problem is split into subproblems and solved iteratively. The choice of dual variables is crucial and plays a dominant role in the computation efforts required.

Singh and Aggarwal [26] considered a short range problem and attempted to solve it through the Dynamic programming technique. It assumes, that the thermal plants are run as base load plant and therefore, depending upon the demand, all the thermal plants will be put into operation one by one and afterwards when the demand exceeds, hydel plants that are run as peak load plants to take care of fluctuating loads will be put into operation. It does not include transmission losses and they are computed for each interval after the schedule has been obtained for a demand corresponding to that interval. This is a serious drawback indeed. Furthermore, the Dynamic programming becomes inefficient when the dimension of the problem is large.

CHAPTER 4

ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEMS INCLUDING TRANSMISSION LOSSES

4.1 INTRODUCTION

The crux of the load scheduling problem is to minimise the total generation cost of the power system while satisfying all the constraints imposed on the system. Several nonlinear programming techniques have been proposed in the literature to solve such problems [3,26,27,29,30]. While they differ in the type of constraints that they can handle and techniques used, the general approach is to solve the problem so as to get a new set of plant generations that reduces the cost of generation and repeat the procedure until no further reduction in the objective function i.e. the cost of generation is possible.

In the present chapter, the formulation and algorithm to solve the short range load scheduling problem of hydro-thermal systems are presented. In a combined hydro-thermal system, there is no (almost negligible) cost of generation associated with hydro stations. However, there is always a constraint on total energy to be generated at hydro stations. The total energy available at each hydro station over any given period of time is determined by the amount of water available for generation. This is why the formulation of hydro-thermal problem can not be an instantaneous problem as one finds the case with purely thermal systems because it is assumed that

the fuel supply to thermal systems is continusously available.

The short range problem attempts to allocate the resources obtained from the solution of long range problem over a day so as to minimise the cost of generation during that day. Long-range problem is not a part of the work reported in this thesis. Hence, in the numerical example, certain amount of water is assumed for utilisation over a day. It is expected that one realises more savings if the transmission line losses are analysed accurately. Transmission losses have been taken into account by loss coefficients developed by Kirchmayer [1,2] The uncertainties in load demand during a day are usually negligible. Hence, it is reasonable to treat load demands to be deterministic. However, the consideration of head variatiom may not be crucial for a short range problem. With this assumption, by discratizing the time into reasonable hourly or half hourly intervals, the short-range can be formulated and solved by the successive linear programming technique after suitably linearising the functions involved about the nominal solution point.

4.2 FORMULATION OF THE PROBLEM

As mentioned earlier, the interval of optimisation (usually a day) is discretized into 'K' equal intervals, such that in each interval the load at all stations remain constant.

It is further assumed that the solution to the long range problem is available, which specifies the hydro energy to be generated at each hydro station. The system that we are going to study is predominantly a thermal system with one or two hydel plants. This is because of our basic assumption that the thermal plants are operated as base load plants and the hydel plants are operated as peak load plants. However, if the number of hydel plants are more than one, they can always be combined into a single equivalent plant on the assumption that each of the hydro-stations are located at about the same distance from the load centre. This is because the incremental transmission loss is about the same. Thus, after obtaining the generation schedule of the equivalent hydel plant, we can assign this generation to the various hydel plants in proportion to their ratings.

Let there be 'N' thermal plants and 'M' hydel plants. Let S_j be the generation of the j^{th} thermal plant and h_i be the generation of the i^{th} hydel plant. The objective function in the present work very closely follows the formulation due to Singh and Aggarwal [26] . Let the cost of generation in any interval for a power (thermal) S_j generated at j^{th} thermal plant be $C_j(S_j)$. If the intervals are not of equal duration, of course, the function C_j^k will be described by different coefficients in each interval, otherwise they do not depend on

the interval. Then the overall cost of generation is given by

$$C_T(S,h,\gamma) = \sum_{k=1}^{K} \sum_{i=1}^{N} C_j(S_j^k) + \sum_{i=1}^{M} \gamma_i \, y_i \, (h_i^k) \quad \text{.. (4.1)}$$

In the Eq. (4.1), C_T is a function of thermal generation S_j, hydel generation h_i and a fictitious cost of water γ_i. While optimising the Eq. (4.1) under the set of constraints, γ_i is made pre-assigned parameter i.e. some appropriate value is assigned to γ_i. The significance of γ_i is to associate a fictitious cost with the hydel generation, otherwise, optimisation process would try to assign all the loads to the hydel plants only. Hence, γ_i controls the generation of the hydel plants, which in turn controls the discharge of the water. The dimension of γ_i is that of water price i.e. Rs/cubic ft. $y_i(h_i)$ is the discharge of water at i^{th} hydel plant in cubic ft hour for a hydel generation of h_i at the i^{th} hydel plant. The higher the value of γ_i, the lesser the discharge of water.

However, if we do not put any constraint in the optimisation of the objective function as defined by the equation (4.1), we would get the trivial solution i.e. zero cost with no generation. Therefore, the following constraints must be imposed.

$$\sum_{j=1}^{N} s_j^k + \sum_{i=1}^{M} h_i^k = P_D^k + P_L(s_j^k, h_i^k) \quad \ldots (4.2)$$

for k=1, ... K

The equation (4.2) states that for any interval of time, the total generation due to hydro as well as thermal generation must be met by the load demand plus losses in that interval. The losses are a function of plant generations. They can be suitably represented by the loss coefficient matrix known as 'B' matrix [1,2].

$$P_L(s_j^k, h_i^k) = \sum_{j=1}^{N} \sum_{i=1}^{N} s_j^k \, b_{TT_{ij}} \, s_i^k + \sum_{j=1}^{M} \sum_{i=1}^{M} h_i^k \, b_{HH_{ij}} \, h_j^k$$

$$+ 2 \sum_{j=1}^{M} \sum_{i=1}^{M} s_j^k \, b_{TH_{ij}} \, h_i^k \quad \ldots (4.3)$$

where $b_{TT_{ij}}$, $b_{HH_{ij}}$ and $b_{TH_{ij}}$ are the elements of the submatrices B_{TT}, B_{HH} and B_{TH} such that the 'B' matrix or the loss coefficient matrix is as under,

$$B = \begin{bmatrix} B_{TT} & B_{TH} \\ B_{TH}^{T} & B_{HH} \end{bmatrix} \quad .. (4.4)$$

Care should be taken that the solution obtained should not dictate generation of any plant beyond the plant capacity or negative generation. This is why the following inequality constraints must also be incorporated

$$S_j^{min} \leq S_j^k \leq S_j^{max} \quad .. (4.5)$$

and $$h_i^{min} \leq h_i^k \leq h_i^{max} \quad .. (4.6)$$

for i=1, ... M

j=1, ... N

and k=1, ... K

However specified amount of water Y_i is discharged at the i^{th} hydel plant over a day. This forms another constraint on the hydel generation h_i at the i^{th} hydel plant.

$$\sum_{k=1}^{K} y_i(h_i^k) = Y_i \quad .. (4.7)$$

In the following section, it will be shown that the problem as formulated here, can be solved by using successive linear programming technique which has the advantages of simplicity in formulation, fast in execution and at the same time, **assurance** of a **solution if** one exists.

4.3 LINEAR PROGRAMMING PROBLEM FORMULATION

The minimisation of the system generation cost given by the equation (4.1) is considered as an objective function. The cost characteristics of the different thermal plants and discharge characteristics of the hydel plants as a function of the plant generations are known. These are non-linear functions and usually represented by a quadratic as shown below,

$$C_j(S_j^k) = a_j + b_j S_j^k + c_j (S_j^k)^2 \qquad .. (4.8)$$

$$y_i(h_i^k) = p_i + q_i h_i^k + r_i (h_i^k)^2 \qquad .. (4.9)$$

where $C_j(S_j^k)$ = cost function of j^{th} thermal plant in Rs/HR

S_j^k = thermal generation of j^{th} thermal plant in MW

$y_i(h_i^k)$ = discharge function of i^{th} hydel plant in Cu.ft./HR

h_i^k = hydel generation of i^{th} hydel plant in MW

and

a_j, b_j, c_j, p_i, q_i and r_i = constants.

The constants in the equations (4.8) and (4.9) are assumed to be known. The objective function of the equation (4.1) can be rewritten as,

$$C_T = \sum_{k=1}^{K} \sum_{j=1}^{N} (a_j + b_j\, S_j^k + c_j (S_j^k)^2)$$

$$+ \sum_{i=1}^{M} \gamma_i (p_i + q_i\, h_i^k + r_i (h_i^k)^2) \qquad .. (4.10)$$

Our approach would be here to minimise the cost of generation in each interval of time over the total optimisation period. While considering the cost associated with a particular interval 'k', we may drop the subscript 'k' from S_j^k and h_i^k and write our objective function for that particular interval as follows

$$f^k = \sum_{j=1}^{N} (a_j + b_j\, S_j + c_j S_j^2) + \sum_{i=1}^{M} {}_i(p_i + q_i\, h_i + r_i\, h_i^2) \qquad .. (4.11)$$

From the equation (4.11), it is evident that C_T in equation (4.10) is nothing but the sum of 'f^k' for all K intervals

over the total optimisation period. We logically develop a feel that the minimum of C_T is the sum of minimum of 'f^k' for all K intervals over the total optimisation period.

Linearising the scalar cost function of equation (4.11) by Taylor series expansion around the initial operating state (S_j^o, h_i^o), the incremental cost function is obtained as,

$$\Delta f^k = \sum_{j=1}^{N} (b_j + 2c_j S_j^o) \, \Delta S_j + \sum_{i=1}^{M} \gamma_i (q_i + 2r_i h_i^o) \, \Delta h_i \qquad .. \; (4.12)$$

i.e.

$$\Delta f^k = \sum_{i=1}^{N+M} d_i \Delta P_i \qquad .. \; (4.13)$$

where d_i is the incremental cost coefficient and ΔP_i is the incremental change in the generation of the plants.

d_i and ΔP_i are given as under

$$d_j = b_j + 2 \, c_j \, S_j^o \qquad .. \; (4.14)$$

for $j=1, \ldots N$

$$d_{N+i} = \gamma_i \, (q_i + 2 \, r_i \, h_i^o) \qquad .. \; (4.15)$$

for $i=1, \ldots M$

$$\Delta P_j = \Delta S_j \quad \text{.. (4.16)}$$

for $j=1, \ldots N$

$$\Delta P_{N+i} = \Delta h_i \quad \text{.. (4.17)}$$

for $i=1, \ldots M$

The constraints of the equations (4.5), (4.6) and (4.2) can be rewritten in the following form, after the dropping the subscript 'k'

$$g_i = S_i - S_i^{min} \geq 0 \quad \text{.. (4.18a)}$$

$$g_{N+i} = S_i^{max} - S_i \geq 0 \quad \text{.. (4.18b)}$$

for $i=1, \ldots N$

$$g_{2N+i} = h_i - h_i^{min} \geq 0 \quad \text{.. (4.19a)}$$

$$g_{2N+M+i} = h_i^{max} - h_i \geq 0 \quad \text{.. (4.19b)}$$

for $i=1, \ldots M$

$$g_{2N+2M+1} = \sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i - P_D - P_L(S_j, h_i) = 0 \quad \text{.. (4.20)}$$

Linearising the equations (4.18), (4.19) and (4.20) around the current operating state by Taylor series expansion, we get the new linearised constraints as follows:

$$g_i = \Delta S_i \geq S_i^{min} - S_i^o \qquad \text{.. (4.21a)}$$

For i=1, ... N

$$g_{N+i} = -\Delta S_i \geq S_i^o - S_i^{max} \qquad \text{.. (4.21b)}$$

for i=1, ... N

$$g_{2N+i} = \Delta h_i \geq h_i^{min} - h_i^o \qquad \text{.. (4.22a)}$$

for i=1, ... M

$$g_{2N+M} = -\Delta h_i \geq h_i^o - S_i^{max} \qquad \text{.. (4.22b)}$$

for i=1, ... M

$$g_{2N+2M+1} = \sum_{j=1}^{N} \left(1-2\sum_{n=1}^{N} S_n^o\, b_{TT_{jn}} - 2\sum_{m=1}^{M} h_m^o\, b_{TH_{mj}}\right)\Delta S_j +$$

$$\sum_{i=1}^{M} \left(1-2\sum_{n=1}^{N} S_n^o\, b_{TH_{ni}} - 2\sum_{m=1}^{M} h_m^o\, b_{HH_{mi}}\right)\Delta h_i =$$

$$P_D + P_L(S_j^o, h_i^o) - \sum_{j=1}^{N} S_j^o - \sum_{i=1}^{M} h_i^o \qquad \text{.. (4.23)}$$

Equation (4.13) together with equations (4.21), (4.22) and (4.23) forms a LP problem in incremental variables ΔS_j and Δh_i. The main requirement of the LP technique is that the problem variables are restricted to be non-negative. But in the formulation proposed here, the incremental variables are unrestricted in sign. To overcome this difficulty Dantzig presents two methods. In one method, the unrestricted variables are expressed as the difference of two non-negative variables This doubles the number of the variables in the problem and leads to increased storage requirement to solve LP problem. Another method is to shift the unrestricted variables, if the minimum or maximum limit of the variables are known, to ensure non-negativity. This reduces the number of constraints because the constraints on minimum or maximum limits for the problem variables are not required. The second method has been preferred in this case because, (i) the limits on the problem variables are known (ii) storage requirement is reduced and (iii) number of constraints gets reduced. Therefore, we define a non-negative vector Z of dimension (N+M) with elements z_i

as follows,

$$z_i = \Delta s_j - \Delta s_j^{min} \qquad (4.24a)$$

for $j=1, \ldots N$

and

$$z_{N+i} = \Delta h_i - \Delta h_i^{min} \qquad .. \quad (4.24b)$$

for $i=1, \ldots M$

where

$$\Delta s_j^{min} = s_j^{min} - s_j^o \qquad .. \quad (4.25a)$$

$$\text{and} \quad \Delta h_i^{min} = h_i^{min} - h_i^o \qquad .. \quad (4.25b)$$

With the variables defined by the equation (4.24), the equations (4.21a) and (4.22a) are redundant. Substitution of equation (4.25) in the equations (4.21b), (4.22b) and (4.23) yields,

$$-z_j \geq s_j^{min} - s_j^{max} \qquad .. \quad (4.26a)$$

for $j=1, \ldots N$

$$-z_{N+i} \geq h_i^{min} - h_i^{max} \qquad .. \quad (4.26b)$$

for $i=1, \ldots M$

$$\sum_{j=1}^{N}\left(1-2\sum_{n=1}^{N} S_n^o\, b_{TT_{jn}} - 2\sum_{m=1}^{M} h_m^o\, b_{TH_{mj}}\right) z_j + \sum_{i=1}^{M}\left(1-2\sum_{n=1}^{N} s_n^o\, b_{TH_{ni}}\right.$$

$$\left.-2\sum_{m=1}^{M} h_m^o\, b_{HH_{mi}}\right) z_{N+i} = P_D - P_L(S_j^o, h_i^o) - \sum_{j=1}^{N}\left(1-2\sum_{n=1}^{N} S_n^o\, b_{TT_{jn}}\right.$$

$$\left.-2\sum_{m=1}^{M} h_m^o\, b_{TH_{mj}}\right) S_j^{min} - \sum_{i=1}^{M}\left(1-2\sum_{n=1}^{N} S_n^o\, b_{TH_{ni}} - 2\sum_{m=1}^{M} h_m^o\, b_{HH_{mi}}\right) h_i^{min}$$

$$\quad .. (4.27)$$

Substitution of equation (4.25) in the expression for incremental objective function given by equation (4.13) yields

$$\Delta f^k = d^T z + d^T \begin{bmatrix} S_j^{min} \\ \\ h_i^{min} \end{bmatrix} \quad .. (4.28)$$

In the equation (4.28), the second term is constant and does not contribute to the optimisation process. The objective function reduces to

$$\Delta f^k = d^T z \quad .. (4.29)$$

Then LP problem can be stated as : Determine Z that minimises the objective function of equation (2.29) subject to the constraints of equations (4.26) and (4.27).

In the present formulation, the number of constraints exceeds the number of variables. Hence dual LP problem is obtained of this primal problem, to attempt the solution because less effort is needed to solve the problem when number of constraints are less than the number of variables. Tables (2.1) and (2.2) in the section (2.2) of Chapter 2 can be used to construct the dual problem.

4.4 SOLUTION PROCEDURE

We shall now describe the solution procedure using dual LP technique for the economic load scheduling problem :

Step 1 : Assume γ_i for all i's. Its choice is crucial. Usually we start with low value such as 0.1.

Step 2 : Assume an initial feasible value of S_j^o and h_i^o. That is, select the initial S_j^o and h_i^o such that it satisfies the condition (4.5) and (4.6)

Step 3 : Set k=1

Step 4 : Set $P_D = P_D^k$

Step 5 : Obtain B and C vectors for primal problem of equation (4.29) by using the R.H.S. of equation (4.26) and (4.27) and equations (4.14) and (4.15) respectively. Formulate the dual LP problem using relations and rules given in the Tables (2.1) and (2.2) of section (2.2) of chapter 2.

Step 6 : Calculate the system cost of generation with S_j^o and h_i^o

Step 7 : Solve the dual LP problem to get corrections i.e. ΔS_j and Δh_i.

Step 8 : Update the S_j and h_i as $S_j^{new} = S_j^o + \Delta S_j$ and $h_i^{new} = h_i^o + \Delta h_i$

Step 9 : Calculate the system cost of generation with S_j^{new} and h_i^{new}.

Step 10: If the difference in the system cost of generation is less than the specified tolerance, store S_j^{new} and h_i^{new} as S_j^k and h_i^k and go to step 11. Otherwise set $S_j^o = S_j^{new}$ and $h_i^o = h_i^{new}$ and go to step 5 and repeat onwards.

Step 11: If k is less than K, set k=k+1, $S_j^o = S_j^k$ and $h_i^o = h_i^k$, and go to step 4 and repeat onwards. Otherwise, go to step 12.

Step 12 : Check for the discharge of water by using equation (4.7). If the discharge is within tolerable band, stop the procedure. Otherwise adjust γ_i and go to step 2 and repeat onwards.

The efficiency of the proposed method greatly depends upon the choice of the water values i.e. γ_i. As already stated, this method is very much powerful for a system which is predominantly a thermal one with one or two hydro-stations. However, they can be combined into one equivalent source for the scheduling purpose, if they are large in number.

4.5 NUMERICAL EXAMPLE

The formulation and the solution procedure discussed in the preceding sections is applied to a short range load scheduling problem of 24 hours duration as the total optimisation interval. The total optimisation interval has been subdivided into equal 24 intervals of one hour each. The demand at the generating station is assumed to be constant during each hourly interval and at the end of each interval, the demand increases or decreases in jumps (or remains constant). The system consists of four generating plants in all, out of them, three are thermal and one, hydel plant. The cost discharge characteristics are

given in Table 4.1.

Table 4.1

Cost/discharge characteristics

Lower bound in MW	Upper bound in MW	Cost characteristic	Quantity of water to be utilised over 24 hour in cubic ft.
		(a) Thermal Plant	
50	200	$C_1=100+0.1S_1+0.01S_1^2$	-
60	170	$C_2=120+0.1S_2+0.02S_1^2$ Rs/hr	-
50	215	$C_3=150+0.2S_3+0.01S_3^2$	-
		(b) Hydel Plant	
15	65	$y_1=140+20h_i+0.06h_i^2$ Cubic ft./hr	18,000

The loss coefficients are taken to be as under.

$$\begin{bmatrix} 0.0005 & 0.00005 & 0.0002 & 0.00003 \\ 0.00005 & 0.00004 & 0.00018 & -0.00011 \\ 0.0002 & 0.00018 & 0.0005 & -0.00012 \\ 0.00003 & -0.00011 & -0.00012 & 0.00023 \end{bmatrix}$$

The above problem was solved on DEC-10 Digital Computer at IIT Kanpur, using the formulation and the solution procedure discussed in the preceding sections for the load demands given in the Table (4.2) for each interval. The results of generation schedule of all the thermal and hydro plants were obtained as shown in Table (4.2). Transmission losses were taken into account by the loss coefficients. The problem converged for a water value of 0.22.

4.6 DISCUSSION

The general formulation proposed in this chapter has shown that the economic load scheduling of Hydro-thermal system can be done by using successive dual LP approach. The equality constraints which are difficult to handle in NLP problems, can be easily handled by this approach.

Since the minimum system generation cost for a particular subinterval is obtained practically in 1 to 2 iterations, this approach is really well suited for large size problems. Number of constraints are sufficiently less than the actual constraints one has to tackle with NLP techniques because no constraints are required for minimum (or maximum) limits for this type of formulation. This results in saving of storage and computational efforts. For a very bad starting

Table 4.2

Results of Generation Schedule

Interval(k)	Demand (P_D^k) in(Mw)	Optimal cost (f^k) in Rs.	Generation Schedule in (Mw) Thermal Plant s_1^k	s_2^k	s_3^k	Hydro-plant h_1^k	Discharge water (in cubic ft)	Transmission losses P_L^k(Mw)
1	175	591.49	55.2	60.0	50.0	15.0	453.5	5.2
2	190	612.82	61.2	60.0	60.5	15.0	453.5	6.7
3	220	665.44	73.6	60.0	81.8	15.0	453.5	10.5
4	280	810.00	107.8	60.4	117.5	15.0	453.5	20.7
5	320	932.19	131.8	79.4	120.3	15.0	453.5	26.5
6	360	1079.45	157.1	81.8	142.7	15.0	453.5	36.7
7	390	1203.80	159.3	96.4	162.4	15.0	453.5	43.1
8	410	1292.86	171.0	107.3	162.4	16.0	474.5	46.7
9	440	1421.63	189.2	109.0	164.5	28.9	767.6	51.6
10	475	1603.60	189.2	134.7	176.9	31.3	824.5	57.1
11	525	1842.16	198.9	134.7	202.4	55.6	1436.2	66.6
12	550	1929.23	200.0	149.0	204.5	65.0	1693.5	68.6
13	565	2080.14	200.0	157.8	215.0	65.0	1693.5	72.8
14	540	1914.92	200.0	139.1	203.0	65.0	1693.5	67.2
15	500	1703.63	189.3	134.7	180.5	52.8	1363.1	57.3
16	450	1471.61	189.2	134.7	176.9	51.3	824.6	57.1
17	425	1347.01	171.2	107.3	164.3	28.9	767.6	46.7
18	400	1248.44	171.0	97.5	162.4	15.0	453.5	46.0
19	375	1138.74	157.1	96.4	144.7	15.0	453.5	38.2
20	340	1003.46	134.6	79.4	142.7	15.0	453.5	31.7
21	300	870.63	110.2	79.4	117.5	15.0	453.5	22.1
22	250	730.17	104.1	60.0	85.9	15.0	453.5	14.9
23	200	628.61	71.1	60.0	61.7	15.0	453.5	7.8
24	180	598.31	60.1	60.0	50.6	15.0	453.5	5.6

Total System cost of generation = Rs.28770.36

Total discharge of water = 17830.7 Cubic ft.

with γ_i as zero, the computer time required to obtain the solution to the numerical problem in section (4.5) is 30 seconds on DEC-10. But for a judicious choice of γ_i as 0.2, it is 3 seconds for the same problem.

Both, the saving in computation efforts and storage make this formulation and algorithm more attractive especially for a large interconnected power system. The proposed method very easily takes the losses into account with the help of loss coefficients due to Kirchmayer [1,2].

CHAPTER 5

CONCLUSION

This chapter is aimed at reviewing the significant results obtained during the course of this work and making a few suggestions for the future line of work in this area. Before reviewing the work done, a brief account of the objectives of this investigation is summarised.

As is already mentioned in chapter 1, the problem of economic load scheduling of hydro-thermal systems is much more complex than that of purely thermal systems. Several attempts have been made in the past to solve the scheduling problem of hydro-thermal systems by using techniques like Dynamic programming [26] and NLP techniques [11,27,29,30]. They have resulted in solution procedures requiring large computational efforts and storage. With the larger interconnected power systems, the procedures become bogged down.

Hence the main objective of the present work has been to find a suitable solution technique which can be applied to the economic load scheduling problem in larger interconnected hydro-thermal systems and which requires lesser computational efforts and storage than the existing methods.

The proposed solution method makes use of the linearised objective function and constraints around a nominal point. In each subinterval, the successive dual LP technique is used

to solve the optimisation problem of hydro-thermal system. Load demands are taken to be deterministic. However the choice of water value γ_i is crucial and the efficiency of the method greatly depends upon it.

The losses have been taken into account by the loss coefficients due to Kirchmayer [1,2]. The load demand equation (4.2) forms the equality constraint and requires to be strictly satisfied. A numerical example of short range scheduling problem has been taken. Based on the investigations made and the results obtained, the following major conclusions can be drawn.

(i) The transmission losses can be taken into account for the load scheduling of hydro-thermal systems.

(ii) Successive dual LP technique can be used to the problem of economic load scheduling.

(iii) Sufficient reduction in the number of constraints, storage and computational efforts can be obtained by the proposed algorithm.

FUTURE SCOPE OF WORK

As a sequel to the results obtained during the present investigation, the following line of future work seems to be worth-pursuing.

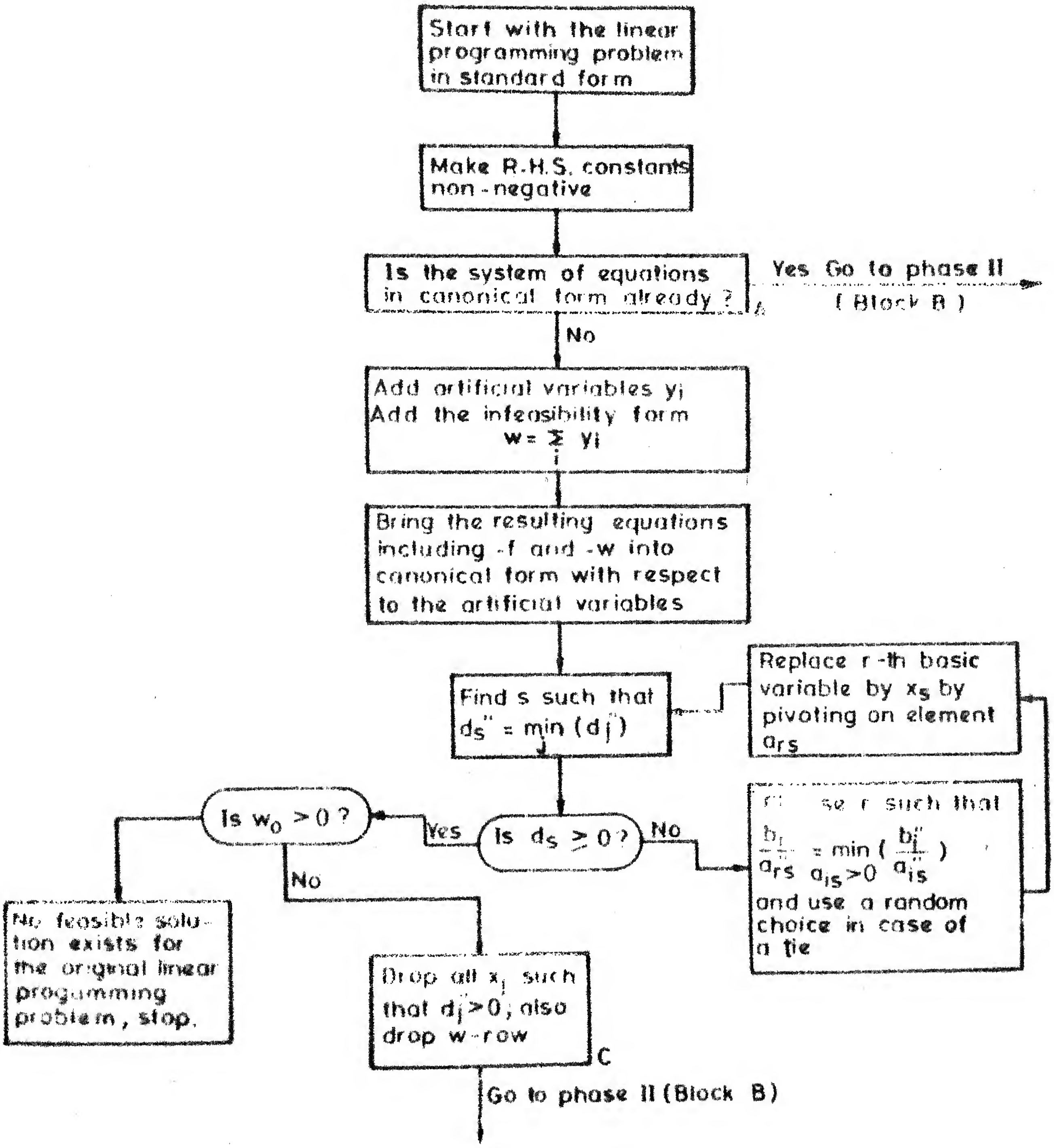
1. For a short range problem, the load demands and the discharge of water are assumed to be deterministic as the uncertainties in them have been ignored. But in actual practice, these can only be characterised probalistically and the scheduling problem becomes a stochastic programming problem. Suitable modifications of the proposed solution procedure may be possible to consider the stochastic nature of the load demands and the discharge of available water.

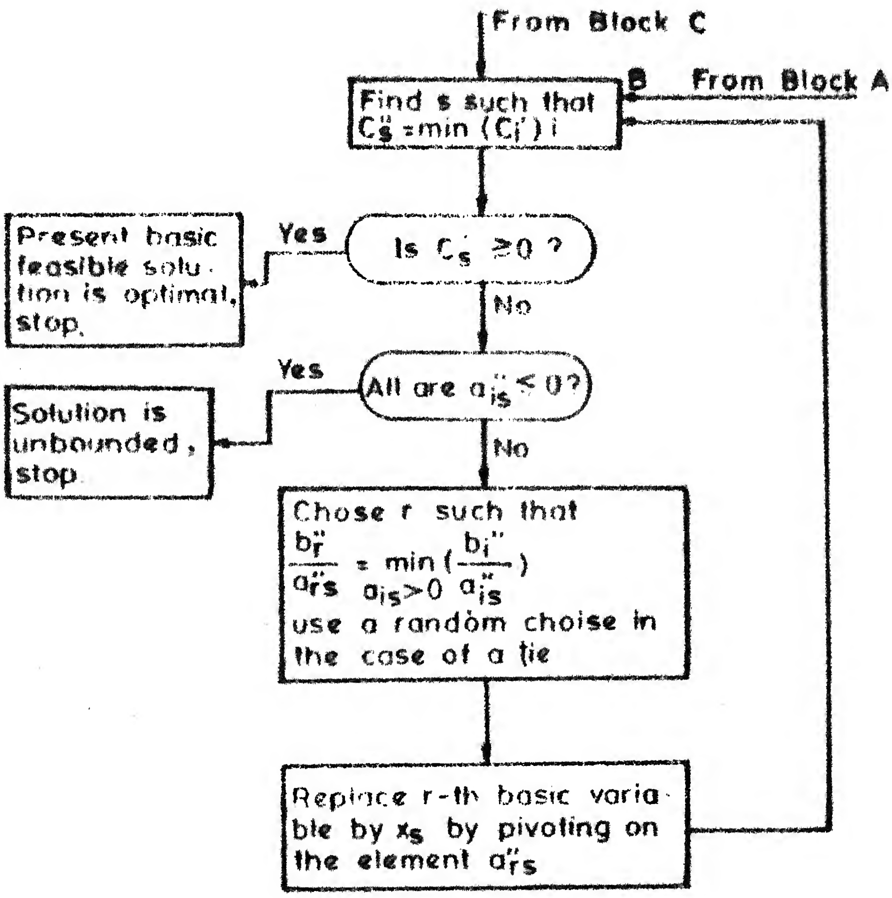
2. A greater saving in the cost can be further achieved by optimising the reactive power of the system. Both, real and reactive generations contribute to the transmission losses. Hence problem should be solved by using A.C. power flow model for the electrical power network. A.C. power flow equations would represent the equality constraints of the electrical network and the operating limits on the voltage magnitudes, reactive powers and line flows etc. may be included in the formulation, in addition to the constraints of the hydro subsystems. Transformer tap-settings should also be included in the model.

3. The head variation of the reservoirs for a short range problem has been ignored in the present work. The dynamics of the reservoirs should be taken into account for the head variations in the optimal scheduling of hydro-thermal systems.

APPENDIX

FLOW DIAGRAM FOR THE TWO-PHASE SIMPLEX METHOD





From Block C
B
From Block A
Find s such that $C_s'' = \min (C_j'')$ i
Is $C_s' \geq 0$?
Yes
Present basic feasible solution is optimal, stop.
No
All are $a_{is}'' \leq 0$?
Yes
Solution is unbounded, stop.
No
Chose r such that
$\frac{b_r''}{a_{rs}''} = \min_{a_{is}>0} (\frac{b_i''}{a_{is}''})$
use a random choise in the case of a tie
Replace r-th basic variable by x_s by pivoting on the element a_{rs}''

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